



## **Dimension Reduction for Single Cell Data Analysis**

Nikolay Oskolkov, Lund University, NBIS SciLifeLab, Sweden scRNAseq course, 01.04.2025





## Brief introduction: who am I



- 2007 PhD in theoretical physics
- 2011 medical genetics at Lund University
- 2016 working at NBIS SciLifeLab, Sweden















# Dimensionality reduction is supposed to ... reduce dimensions



The goal of dimension reduction is not only visualization but also reducing dimensions

## Literature on the Curse of Dimensionality

![](_page_4_Picture_1.jpeg)

## The curse(s) of dimensionality

There is such a thing as too much of a good thing.

### Naomi Altman and Martin Krzywinski

e generally think that more information is better than less. However, in the 'big data' era, the sheer number of variables that can be collected from a single sample can be problematic. This embarrassment of riches is called the 'curse of dimensionality'<sup>1</sup> (CoD) and manifests itself in a variety of ways. This month, we discuss four important problems of dimensionality as it applies to data sparsity<sup>1,2</sup>, multicollinearity<sup>3</sup>, multiple testing<sup>4</sup> and overfitting<sup>5</sup>. These effects are amplified by poor data quality, which may increase with the number of variables.

Throughout, we use *n* to indicate the sample size from the population of interest and *p* to indicate the number of observed variables, some of which may have missing values for some samples. For example, we may have n = 1,000 subjects and p = 200,000 single-nucleotide polymorphisms (SNPs). First, as the dimensionality *p* increases,

First, as the dimensionality *p* increases, the 'volume' that the samples may occupy

![](_page_4_Figure_8.jpeg)

#### Fig. 1 | Data tend to be sparse in higher

**dimensions.** Among 1,000 (*x*, *y*) points in which both *x* and *y* are normally distributed with a mean of 0 and s.d.  $\sigma$  = 1, only 6% fall within  $\sigma$  of (*x*, *y*) = (1.5, 1.5) (blue circle). However, when the data are projected into a lower dimension—shown by histograms—about 30% of the points (all bins A and 100 to have the minor allele a. If we tabulate on two SNPs, A and B, we will expect only ten samples to exhibit both minor alleles with genotype ab. With SNPs A, B and C, we expect only one sample to have genotype abc, and with four or more SNPs, we expect empty cells in our table. We need a much larger sample size to observe samples with all the possible genotypes. As *p* increases, we may quickly find that there are no samples with similar values of a predictor.

Even with just five SNPs, our ability to predict and classify the samples is impeded because of the small number of subjects that have similar genotypes. In situations where there are many gene variants, this effect is exacerbated, and it may be very difficult to find affected subjects with similar genotypes and hence to predict or classify on the basis of genetic similarity.

If we treat the distance between points (e.g., Euclidian distance) as a measure of similarity, then we interpret greater distance

Altman N, Krzywinski M. The curse(s) of dimensionality. Nat Methods. 2018 Jun;15(6):399-400. doi: 10.1038/s41592-018-0019-x. PMID: 29855577.

![](_page_4_Figure_15.jpeg)

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**Fig. 3 | The number of false positives increases with each additional predictor.** The box plots show the number of false positive regression-fit *P* values (tested at  $\alpha = 0.05$ ) of 100 simulated multiple regression fits on various numbers of samples (n = 100, 250 and 1,000) in the presence of one true predictor and k = 10 and 50 extraneous uncorrelated predictors. Box plots show means (black center lines), 25th and 75th percentiles (box edges), and minimum and maximum values (whiskers). Outliers (dots) are jittered.

### Correcting for multiple testing does not solve the problem of too many false-positive hits

![](_page_5_Picture_0.jpeg)

![](_page_5_Picture_1.jpeg)

## Dimension reduction techniques: linear vs. non-linear

![](_page_6_Picture_0.jpeg)

## Linear dimensionality reduction

![](_page_6_Picture_2.jpeg)

![](_page_6_Figure_3.jpeg)

M.Bartoschek, N. Oskolkov et al., Nature Communications 2018

## Non-linear dimensionality reduction

NB

![](_page_7_Picture_1.jpeg)

![](_page_7_Figure_2.jpeg)

M.Bartoschek, N. Oskolkov et al., Nature Communications 2018

![](_page_8_Picture_0.jpeg)

![](_page_8_Picture_1.jpeg)

# Linear dimensionality reduction (MDS, PCA)

![](_page_9_Figure_0.jpeg)

## Estimating the number of informative PCs

•\]

NR

![](_page_10_Figure_1.jpeg)

### **NB**S PCA golden standard in PopGen: still criticized SciLifeLab

![](_page_11_Figure_1.jpeg)

### scientific reports

### **Principal Component Analyses** (PCA)-based findings in population genetic studies are highly biased and must be reevaluated

www.nature.com/scientificreports

Check for updates

Eran Elhaik

Principal Component Analysis (PCA) is a multivariate analysis that reduces the complexity of datasets while preserving data covariance. The outcome can be visualized on colorful scatterplots, ideally with only a minimal loss of information. PCA applications, implemented in well-cited packages like EIGENSOFT and PLINK, are extensively used as the foremost analyses in population genetics and related fields (e.g., animal and plant or medical genetics). PCA outcomes are used to shape study design, identify, and characterize individuals and populations, and draw historical and ethnobiologica conclusions on origins, evolution, dispersion, and relatedness. The replicability crisis in science has prompted us to evaluate whether PCA results are reliable, robust, and replicable. We analyzed twelve common test cases using an intuitive color-based model alongside human population data. We demonstrate that PCA results can be artifacts of the data and can be easily manipulated to generate desired outcomes. PCA adjustment also yielded unfavorable outcomes in association studies. PCA results may not be reliable, robust, or replicable as the field assumes. Our findings raise concerns about the validity of results reported in the population genetics literature and related fields that place a disproportionate reliance upon PCA outcomes and the insights derived from them. We conclude that PCA may have a biasing role in genetic investigations and that 32,000-216,000 genetic studies should be reevaluated. An alternative mixed-admixture population genetic model is discussed.

The ongoing reproducibility crisis, undermining the foundation of science<sup>1</sup>, raises various concerns ranging from study design to statistical rigor2.3. Population genetics is confounded by its utilization of small sample sizes, ignorance of effect sizes, and adoption of questionable study designs. The field is relatively small and may involve financial interests<sup>4-6</sup> and ethical dilemmas<sup>7,8</sup>. Since biases in the field rapidly propagate to related disciplines like medical genetics, biogeography, association studies, forensics, and paleogenomics in humans and non-humans alike, it is imperative to ask whether and to what extent our most elementary tools satisfy risk criteria.

Principal Component Analysis (PCA) is a multivariate analysis that reduces the data's dimensionality while preserving their covariance. When applied to genotype bi-allelic data, typically encoded as AA, AB, and BB, PCA finds the eigenvalues and eigenvectors of the covariance matrix of allele frequencies. The data are reduced to a small number of dimensions termed principal components (PCs); each describes a decreased proportion of the genomic variation. Genotypes are then projected onto space spanned by the PC axes, which allows visualizing the samples and their distances from one another in a colorful scatter plot. In this visualization, sample overlap is considered evidence of identity, due to common origin or ancestry<sup>9,10</sup>. PCA's most attractive property for population geneticists is that the distances between clusters allegedly reflect the genetic and geographic distance between them. PCA also supports the projection of points onto the components calculated by a different dataset. presumably accounting for insufficient data in the projected dataset. Initially adapted for human genomic data in 196311, the popularity of PCA has slowly increased over time. It was not until the release of the SmartPCA tool (EIGENSOFT package)10 that PCA was propelled to the front stage of population genetics.

PCA is used as the first analysis of data investigation and data description in most population genetic analyses, e.g., Refs.12-15. It has a wide range of applications. It is used to examine the population structure of a cohort or individuals to determine ancestry, analyze the demographic history and admixture, decide on the genetic similarity of samples and exclude outliers, decide how to model the populations in downstream analyses, describe the ancient and modern genetic relationships between the samples, infer kinship, identify ancestral clines in the data, e.g., Refs.<sup>16-19</sup>, detect genomic signatures of natural selection, e.g., Ref.<sup>20</sup> and identify convergent evolution<sup>2</sup>

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![](_page_12_Picture_0.jpeg)

## PCA has a known pitfall: uneven sampling of populations

![](_page_12_Picture_2.jpeg)

![](_page_12_Figure_3.jpeg)

**Downsampled Europeans** 

**Downsampled Asians** 

![](_page_13_Picture_0.jpeg)

## PCA for Single Cell applications

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3k Peripheral Blood Mononuclear Cells (PBMC) available from 10X Genomics

![](_page_13_Figure_4.jpeg)

Two principal components (PCs) seem to be insufficient to fully reveal heterogeneity in single cell gene expression data.

Solution: use more PCs or tSNE / UMAP

https://satijalab.org/seurat/articles/pbmc3k\_tutorial.html

## NBS PCA vs. tSNE: when data complexity grows SciLifeLab

### Three classes of data points

![](_page_14_Figure_2.jpeg)

### PCA and tSNE tell the same story

### Ten classes of data points

![](_page_14_Figure_5.jpeg)

tSNE is more informative than PCA

Oskolkov et al., unpublished

### PCA works fine on a linear manifold

![](_page_15_Picture_1.jpeg)

0.1

5

10

0.2

0.3

0.4

20

15

![](_page_15_Figure_2.jpeg)

NR

![](_page_16_Picture_0.jpeg)

### PCA vs. tSNE vs. UMAP on non-linear manifold

![](_page_16_Picture_2.jpeg)

![](_page_16_Figure_3.jpeg)

![](_page_16_Figure_4.jpeg)

![](_page_16_Figure_5.jpeg)

![](_page_16_Figure_6.jpeg)

## Why PCA can't unwrap the Swiss Roll SciLifeLab

![](_page_17_Picture_1.jpeg)

#### MDS Linkages

LLE Linkages (100 NN)

![](_page_17_Figure_5.jpeg)

![](_page_17_Figure_6.jpeg)

![](_page_17_Figure_7.jpeg)

![](_page_18_Picture_0.jpeg)

![](_page_18_Picture_1.jpeg)

# Nonlinear dimensionality reduction (tSNE, UMAP)

![](_page_19_Picture_0.jpeg)

## Non-linear dimension reduction: neighborhood graph

![](_page_19_Picture_2.jpeg)

1) Construct high-dimensional graph

![](_page_19_Figure_4.jpeg)

3) Collapse the graphs together

Kullback-Leibler divergence

![](_page_20_Picture_0.jpeg)

Compute low

dimensional affinities

Construct matrix Q

### tSNE dimension reduction algorithm

Compute cost function

![](_page_20_Figure_2.jpeg)

-40

-20

tSNF1

0

20

40

![](_page_21_Picture_0.jpeg)

## How to select optimal perplexity

![](_page_21_Picture_2.jpeg)

Van der Maaten: "Loosely speaking, one could say that a larger / denser dataset requires a larger perplexity."

![](_page_21_Figure_4.jpeg)

PERPLEXITY VS. NUMBER OF CELLS: LOGARITHMIC SCALE

 $ext{Perplexity} \sim N^{rac{1}{2}}$ 

## **NB**S Limitations of tSNE and promise of UMAP • SciLifeLab

tSNE does not scale for large data sets?

tSNE does not preserve global structure?

tSNE can only embed into 2-3 dims?

tSNE performs non-parametric mapping (no variance explained statistics)?

tSNE can not work with high-dimensional data directly (PCA needed)?

tSNE uses too much RAM at large perp?

![](_page_22_Figure_7.jpeg)

![](_page_23_Picture_0.jpeg)

## How is UMAP different from tSNE

UMAP uses local connectivity for high-dim probabilities

UMAP does not normalize probabilities (speed-up)

UMAP can deliver a number of components for clustering

UMAP uses Laplacian Eigenmap for initialization

UMAP uses Cross-Entropy (not KL) as cost function

$$CE(X,Y) = \sum_{i} \sum_{j} \left[ p_{ij}(X) \log\left(\frac{p_{ij}(X)}{q_{ij}(Y)}\right) + (1 - p_{ij}(X)) \log\left(\frac{1 - p_{ij}(X)}{1 - q_{ij}(Y)}\right) \right]$$
  
This is similar to tSNE cost function This term is UMAP specific

JMAP2

![](_page_23_Figure_8.jpeg)

![](_page_24_Picture_0.jpeg)

![](_page_24_Picture_1.jpeg)

## tSNE vs. UMAP: global structure preservation

## **NBES** Cost function seems to make UMAP SciLifeLab preserve more of global structure than tSNE

![](_page_25_Figure_1.jpeg)

 $X \rightarrow$  infinity, Y can be any

 $X \rightarrow$  infinity,  $Y \rightarrow$  infinity

## Why preserving global structure is important

![](_page_26_Figure_1.jpeg)

Can large perplexity solve the problem of global structure for tSNE?

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## KL-gradient goes to zero at large perplexity SciLifeLab

![](_page_27_Figure_1.jpeg)

## BES tSNE degrades to PCA on non-linear manifold at large perplexity SciLifeLab

![](_page_28_Figure_1.jpeg)

### 

Laplacian

![](_page_29_Figure_1.jpeg)

MNIST LAPLACIAN EIGENMAP

![](_page_29_Figure_3.jpeg)

![](_page_29_Figure_4.jpeg)

 $L = I - D^{-1} * S$ 

![](_page_29_Figure_5.jpeg)

Moon et al., Nat Biotechnol. 2019; 37(12):1482-1492

PHATE PLOT

![](_page_29_Figure_8.jpeg)

![](_page_30_Picture_0.jpeg)

![](_page_30_Picture_1.jpeg)

## Autoencoders for dimension reduction of single cell data

![](_page_31_Figure_0.jpeg)

### Application of autoencoder for single cell

![](_page_32_Figure_1.jpeg)

![](_page_33_Picture_0.jpeg)

![](_page_33_Picture_1.jpeg)

# Variance explained by PCA, tSNE and UMAP

![](_page_34_Picture_0.jpeg)

## Variance explained by PCA components

![](_page_34_Picture_2.jpeg)

![](_page_34_Figure_3.jpeg)

![](_page_35_Picture_0.jpeg)

## Variance explained by UMAP components

![](_page_35_Picture_2.jpeg)

![](_page_35_Figure_3.jpeg)

![](_page_36_Picture_0.jpeg)

![](_page_36_Picture_1.jpeg)

## **UMAP in Population Genomics**

![](_page_37_Picture_0.jpeg)

## UMAP: Single Cell vs. PopGen

![](_page_37_Figure_2.jpeg)

![](_page_37_Figure_3.jpeg)

![](_page_38_Picture_0.jpeg)

## **UMAP** (and Single Cell) Criticism

![](_page_38_Picture_2.jpeg)

#### PLOS COMPUTATIONAL BIOLOGY

#### PERSPECTIVE

![](_page_38_Picture_5.jpeg)

### The specious art of single-cell genomics

#### Tara Chario<sup>1</sup>. Lior Pachter<sup>1,2</sup>\*

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![](_page_38_Picture_10.jpeg)

OPEN ACCESS

pcbi.101128

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Dimensionality reduction is standard practice for filtering noise and identifying relevant features in large-scale data analyses. In biology, single-cell genomics studies typically begin with reduction to 2 or 3 dimensions to produce "all-in-one" visuals of the data that are amenable to the human eye, and these are subsequently used for qualitative and quantitative exploratory analysis. However, there is little theoretical support for this practice, and we show that extreme dimension reduction, from hundreds or thousands of dimensions to 2. inevitably induces significant distortion of high-dimensional datasets. We therefore examine the practical implications of low-dimensional embedding of single-cell data and find that extensive distortions and inconsistent practices make such embeddings counter-productive for exploratory, biological analyses. In lieu of this, we discuss alternative approaches for conducting targeted embedding and feature exploration to enable hypothesis-driven biological discovery.

The high-dimensionality of "big data" genomics datasets has led to the ubiquitous application

of dimensionality reduction to filter noise, enable tractable computation, and to facilitate

exploratory data analysis (EDA). Ostensibly, the goal of this reduction is to preserve and

extract local and/or global structures from the data for biological inference [1-3]. Trial and

error application of common techniques has resulted in a currently popular workflow combin-

ing initial dimensionality reduction to a few dozen dimensions, often using principal compo-

UMAP [1,2,5,6]. For single-cell genomics in particular, these embeddings are used extensively

in qualitative and quantitative EDA tasks that fall into 4 main categories of applications (Fig 1,

Embeddings are used to visually assess the extent of integration, mixing, or similarities

between cells from different batches [7-9] and to compare methods of integration/batch-cor-

rection [10]. For guery dataset(s) mapped onto reference datasets/embeddings, visuals likewise

nent analysis (PCA), with further nonlinear reduction to 2 dimensions using t-SNE [4] or

· Modality-mixing, integration, and reference mapping:

· Cluster validation and relationships:

provide an assessment of merged data similarities or differences [11,12].

#### Copyright: @ 2023 Chari, Pachter, This is an open access article distributed under the terms of the Introduction

"Application"):

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19(8); e1011288, https://doi.org/10.1371/journal

Editor: Jason A. Papin, University of Virginia.

Data Availability Statement: Download links for the original data used to generate the figures and results in the paper are listed in Table A in S1 Text. Processed and normalized versions of the count matrices are available on CaltechData, with links provided in Table B in S1 Text. All analysis code used to generate the figures and results in the paper is available at https://github.com/pachterlab CP\_2023 and deposited at Zenodo (DOI https://doi org/10.5281/zenodo.8087950). Code is provided in Colab notebooks which can be run for free on the Google cloud

Funding: L.P. received the National Institutes of Health (nih.gov) award U19MH114830. administered by the National Institute of Menta Health (nimh.nih.gov). T.C. and L.P. were partially

#### Post

Jonathan Pritchard

O Hispanic or Latino

1/20

Not Hispanic or Latino O No information

It's a pity that All of Us used UMAP to visualize ancestry variation in their new marker paper, out today in Nature.

The UMAP algorithm, by design, exaggerates the distinctiveness of the most frequent ancestries, a message that can be misinterpreted by the public.

![](_page_38_Figure_21.jpeg)

### **Biologists, stop putting** UMAP plots in your papers

UMAP is a powerful tool for exploratory data analysis, but without a clear understanding of how it works, it can easily lead to confusion and misinterpretation.

SCHOOL OF PUBLIC HEALTH

Primary Faculty

Professor of Biostatistics

Department of Biostatistic

Rafael A. Irizarry

Disstatistics, Hanvard TH, Chan School of Public Healt

Home / Faculty and Researcher Profiles / Rafael A, Irizarry

HARVARD

TH CHAN

AUTHOR Rafael Irizarry DURUSHED

хI

population 9.216

Dec. 23, 2024

library(Matrix) library(ggplot2) library(dplyr) library(umap) set.seed(2024-6-21) load("rda/pop gen sample.RData")

### The UMAP craze in singe cell RNA-Seq

![](_page_38_Picture_28.jpeg)

#### PLOS Computational Biology | https://doi.org/10.1371/journal.pcbi.1011288 August 17, 2023

Abstract

![](_page_39_Picture_0.jpeg)

### Does UMAP make artificial clusters?

The issue becomes more significant when the underlying mathematics of UMAP is not fully understood. UMAP takes a *p*-dimensional vector of numeric values, such as gene expression in scRNA-Seq, and applies a mathematical transformation to produce two values, resulting in the two coordinates shown in the plot. But what exactly is this function? Do the authors who include these plots in papers fully understand the mathematics behind it? What genes are included in the calculation and how? How exactly does distance in the two dimensional summary relate to the actual distance in *p*-dimensional space? The actual summary function is rarely if ever explained, leaving readers uncertain about what the plot truly represents.

Additionally, UMAP is highly sensitive and can create separations in data that shouldn't necessarily exist. For example, consider applying UMAP to 100 randomly generated points from a multivariate normal distribution representing three correlated random variables:

Sigma <- matrix(.8, 3, 3); diag(Sigma) <- 1
x <- MASS::mvrnorm(100, rep(0,3), Sigma)
#x <- matrix(rnorm(100), ncol = 1)
u <- umap(as.matrix(dist(x)))
ranks <- rank(rowHeans(x))
colors <- colorRampPalette(c("blue", "red"))(nrow(x))
colormap <- colors[ranks]
plot(u\$layout[,1], u\$layout[,2], type = "n", xlab = "dim1", ylab = "dim2")
text(u\$layout[,1], u\$layout[,2], labels = ranks, col = colormap, cex = 0.5)</pre>

![](_page_39_Figure_5.jpeg)

![](_page_39_Figure_6.jpeg)

![](_page_39_Figure_7.jpeg)

this is the output (with "dist" on the left, without "dist" on the right) Rafael Irizarry @rafalab · 1h My recollection is that the version I was using took distance as input Maybe I was wrong. So I updated to the latest, changed code to explicitly tell UMAP the input is a distance matrix, clarify that not every simulation results in separation & thank you in the acknolwedgements Nikolay Oskolkov @NikolayOskolkov · 10h Regarding your code for demonstrating artificial separation of data points, may I ask about the motivation to compute the distance matrix here "u<-umap(as.matrix(dist(x)))"? Are you using 3-dimensional or 100dimensional data? In the code above you input 100-dimensional data

	x <- MASS::mvrnorm(100, rep(0,3), Sigma)
Г	custom.settings <- umap.defaults
	custom.settings\$input <- "dist"
Ŀ	u <- umap(as.matrix(dist(x)), config = custom.settings)
	ranks <- rank(rowMeans(x))
	<pre>colors &lt;- colorRampPalette(c("blue", "red"))(nrow(x))</pre>
	colormap <- colors[ranks]
	<pre>plot(u\$layout[,1], u\$layout[,2], type = "n", xlab = "dim1", ylab = "dim2")</pre>
	<pre>text(u\$layout[,1], u\$layout[,2], labels = ranks, col = colormap, cex = 0.5</pre>
-	

![](_page_40_Figure_0.jpeg)

• Because of their meaningless inter-cluster distances tSNE / UMAP are less useful for population genomics than PCA.

- The goal of tSNE / UMAP is to **discover clusters**, which is sufficient for Single Cell Biology but not for PopGen.
- In PopGen we generally do not discover clusters, we have an idea about e.g. human populations, and the aim is often to explore the **genetic relatedness** between the populations, a task UMAP can absolutely not solve!

![](_page_41_Picture_0.jpeg)

## National Bioinformatics Infrastructure Sweden (NBIS)

![](_page_41_Picture_2.jpeg)

![](_page_41_Picture_3.jpeg)

Knut och Alice Wallenbergs Stiftelse

![](_page_41_Picture_5.jpeg)

![](_page_41_Picture_6.jpeg)

Vetenskapsrådet

LUNDS UNIVERSITET