J.1 Table of Critical Values for the Wilcoxon Rank-Sum Test

The tables on the following pages provide critical values for the Wilcoxon rank-sum test for independent samples with sizes from 3 to 25. Column m is the sample size for the smaller sample and column n is the sample size for the larger sample. If the sample sizes are equal, either sample can be designated m. For each pair of sample sizes (m, n) there are two sets of critical values, one set for one-tail $\alpha = 0.025$ and two-tail $\alpha = 0.05$ and a second set for one-tail $\alpha = 0.05$ and two-tail $\alpha = 0.10$. Suppose for a two-tailed test at $\alpha = 0.05$ we have $m = 8$ and $n = 9$. In the appropriate row and column we find the following numbers 51 93 16 0.0232. The 51 and 93 are the lower and upper critical values for W_X, the statistic testing H_0: $M_X = M_Y$. If $W_X \leq 51$ or $W_X \geq 93$, H_0 would be rejected. The value 0.0232 is the exact P value for the critical values of 51 or 93. The 16 under the column heading d is called the depth. Basically d is the depth one must go into the rank-orederd elementary estimates from each end to find the confidence limit values. In this case, the 16th smallest elementary estimate and the 16th largest elementary estimate are the 95% confidence interval limits for $M_x - M_y$.

1-tail		$\alpha = 0.025$			$\alpha = 0.05$		
2-tail		$\alpha = 0.05$			$\alpha = 0.10$		
m	n	W	d	P	W	d	P
3	3				6 15	1	.0500
3	4				6 18	1	.0286
3	5	6 21	1	.0179	7 20	2	.0357
3	6	7 23	2	.0238	8 22	3	.0476
3	7	7 26	2	.0167	8 25	3	.0333
3	8	8 28	3	.0242	9 27	4	.0424
3	9	8 31	3	.0182	10 29	5	.0500
3	10	9 33	4	.0245	10 32	5	.0385
3	11	9 36	4	.0192	11 34	6	.0440
3	12	10 38	5	.0242	11 37	6	.0352
3	13	10 41	5	.0196	12 39	7	.0411
3	14	11 43	6	.0235	13 41	8	.0456
3	15	11 46	6	.0196	13 44	8	.0380
3	16	12 48	7	.0237	14 46	9	.0423
3	17	12 51	7	.0202	15 48	10	.0465
3	18	13 53	8	.0233	15 51	10	.0398
3	19	13 56	8	.0201	16 53	11	.0435
3	20	14 58	9	.0232	17 55	12	.0469
3	21	14 61	9	.0203	17 58	12	.0410
3	22	15 63	10	.0230	18 60	13	.0443
3	23	15 66	10	.0204	19 62	14	.0473
3	24	16 68	11	.0229	19 65	14	.0421
3	25	16 71	11	.0205	20 67	15	.0449
4	4	10 26	1	.0143	11 25	2	.0286
4	5	11 29	2	.0159	12 28	3	.0317
4	6	12 32	3	.0190	13 31	4	.0333
4	7	13 35	4	.0212	14 34	5	.0364
4	8	14 38	5	.0242	15 37	6	.0364
4	9	14 42	5	.0168	16 40	7	.0378
4	10	15 45	6	.0180	17 43	8	.0380
4	11	16 48	7	.0198	18 46	9	.0388
4	12	17 51	8	.0209	19 49	10	.0390
4	13	18 54	9	.0223	20 52	11	.0395
4	14	19 57	10	.0232	21 55	12	.0395
4	15	20 60	11	.0243	22 58	13	.0400
4	16	21 63	12	.0250	24 60	15	.0497
4	17	21 67	12	.0202	25 63	16	.0493
4	18	22 70	13	.0212	26 66	17	.0491
4	19	23 73	14	.0219	27 69	18	.0487
4	20	24 76	15	.0227	28 72	19	.0485
4	21	25 79	16	.0233	29 75	20	.0481
4	22	26 82	17	.0240	30 78	21	.0480
4	23	27 85	18	.0246	31 81	22	.0477
4	24	27 89	18	.0211	32 84	23	.0475
4	25	28 92	19	.0217	33 87	24	.0473
5	5	17 38	3	.0159	19 36	5	.0476
5	6	18 42	4	.0152	20 40	6	.0411
5	7	20 45	6	.0240	21 44	7	.0366
5	8	21 49	7	.0225	23 47	9	.0466
5	9	22 53	8	.0210	24 51	10	.0415

1-tail		$\alpha = 0.025$			$\alpha = 0.05$		
2-tail		$\alpha = 0.05$			$\alpha = 0.10$		
m	n	W	d	P	W	d	P
5	10	23 57	9	.0200	26 54	12	.0496
5	11	24 61	10	.0190	27 58	13	.0449
5	12	26 64	12	.0242	28 62	14	.0409
5	13	27 68	13	.0230	30 65	16	.0473
5	14	28 72	14	.0218	31 69	17	.0435
5	15	29 76	15	.0209	33 72	19	.0491
5	16	30 80	16	.0201	34 76	20	.0455
5	17	32 83	18	.0238	35 80	21	.0425
5	18	33 87	19	.0229	37 83	23	.0472
5	19	34 91	20	.0220	38 87	24	.0442
5	20	35 95	21	.0212	40 90	26	.0485
5	21	37 98	23	.0243	41 94	27	.0457
5	22	38 102	24	.0234	43 97	29	.0496
5	23	39 106	25	.0226	44 101	30	.0469
5	24	40 110	26	.0219	45 105	31	.0445
5	25	42 113	28	.0246	47 108	33	.0480
6	6	26 52	6	.0206	28 50	8	.0465
6	7	27 57	7	.0175	29 55	9	.0367
6	8	29 61	9	.0213	31 59	11	.0406
6	9	31 65	11	.0248	33 63	13	.0440
6	10	32 70	12	.0210	35 67	15	.0467
6	11	34 74	14	.0238	37 71	17	.0491
6	12	35 79	15	.0207	38 76	18	.0415
6	13	37 83	17	.0231	40 80	20	.0437
6	14	38 88	18	.0204	42 84	22	.0457
6	15	40 92	20	.0224	44 88	24	.0474
6	16	42 96	22	.0244	46 92	26	.0490
6	17	43 101	23	.0219	47 97	27	.0433
6	18	45 105	25	.0236	49 101	29	.0448
6	19	46 110	26	.0214	51 105	31	.0462
6	20	48 114	28	.0229	53 109	33	.0475
6	21	50 118	30	.0244	55 113	35	.0487
6	22	51 123	31	.0224	57 117	37	.0498
6	23	53 127	33	.0237	58 122	38	.0452
6	24	54 132	34	.0219	60 126	40	.0463
6	25	56 136	36	.0231	62 130	42	.0473
7	7	36 69	9	.0189	39 66	12	.0487
7	8	38 74	11	.0200	41 71	14	.0469
7	9	40 79	13	.0209	43 76	16	.0454
7	10	42 84	15	.0215	45 81	18	.0439
7	11	44 89	17	.0221	47 86	20	.0427
7	12	46 94	19	.0225	49 91	22	.0416
7	13	48 99	21	.0228	52 95	25	.0484
7	14	50 104	23	.0230	54 100	27	.0469
7	15	52 109	25	.0233	56 105	29	.0455
7	16	54 114	27	.0234	58 110	31	.0443
7	17	56 119	29	.0236	61 114	34	.0497
7	18	58 124	31	.0237	63 119	36	.0484
7	19	60 129	33	.0238	65 124	38	.0471
7	20	62 134	35	.0239	67 129	40	.0460

1-tail		$\alpha = 0.025$				$\alpha = 0.05$				1-tail		$\alpha = 0.025$				$\alpha = 0.05$			
2-tail		$\alpha = 0.05$				$\alpha = 0.10$				2-tail		$\alpha = 0.05$				$\alpha = 0.10$			
m	n	W		d	P	W		d	P	m	n	W		d	P	W		d	P
7	21	64	139	37	.0240	69	134	42	.0449	10	20	110	200	56	.0245	117	193	62	.0498
7	22	66	144	39	.0240	72	138	45	.0492	10	21	113	207	59	.0241	120	200	65	.0478
7	23	68	149	41	.0241	74	143	47	.0481	10	22	116	214	62	.0237	123	207	68	.0459
7	24	70	154	43	.0241	76	148	49	.0470	10	23	119	221	65	.0233	127	213	72	.0482
7	25	72	159	45	.0242	78	153	51	.0461	10	24	122	228	68	.0230	130	220	75	.0465
8	8	49	87	14	.0249	51	85	16	.0415	10	25	126	234	72	.0248	134	226	79	.0486
8	9	51	93	16	.0232	54	90	19	.0464	11	11	96	157	31	.0237	100	153	34	.0440
8	10	53	99	18	.0217	56	96	21	.0416	11	12	99	165	34	.0219	104	160	38	.0454
8	11	55	105	20	.0204	59	101	24	.0454	11	13	103	172	38	.0237	108	167	42	.0467
8	12	58	110	23	.0237	62	106	27	.0489	11	14	106	180	41	.0221	112	174	46	.0477
8	13	60	116	25	.0223	64	112	29	.0445	11	15	110	187	45	.0236	116	181	50	.0486
8	14	62	122	27	.0211	67	117	32	.0475	11	16	113	195	48	.0221	120	188	54	.0494
8	15	65	127	30	.0237	69	123	34	.0437	11	17	117	202	52	.0235	123	196	57	.0453
8	16	67	133	32	.0224	72	128	37	.0463	11	18	121	209	56	.0247	127	203	61	.0461
8	17	70	138	35	.0247	75	133	40	.0487	11	19	124	217	59	.0233	131	210	65	.0468
8	18	72	144	37	.0235	77	139	42	.0452	11	20	128	224	63	.0244	135	217	69	.0474
8	19	74	150	39	.0224	80	144	45	.0475	11	21	131	232	66	.0230	139	224	73	.0480
8	20	77	155	42	.0244	83	149	48	.0495	11	22	135	239	70	.0240	143	231	77	.0486
8	21	79	161	44	.0233	85	155	50	.0464	11	23	139	246	74	.0250	147	238	81	.0490
8	22	81	167	46	.0223	88	160	53	.0483	11	24	142	254	77	.0237	151	245	85	.0495
8	23	84	172	49	.0240	90	166	55	.0454	11	25	146	261	81	.0246	155	252	89	.0499
8	24	86	178	51	.0231	93	171	58	.0472	12	12	115	185	38	.0225	120	180	42	.0444
8	25	89	183	54	.0247	96	176	61	.0488	12	13	119	193	42	.0229	125	187	47	.0488
9	9	62	109	18	.0200	66	105	22	.0470	12	14	123	201	46	.0232	129	195	51	.0475
9	10	65	115	21	.0217	69	111	25	.0474	12	15	127	209	50	.0234	133	203	55	.0463
9	11	68	121	24	.0232	72	117	28	.0476	12	16	131	217	54	.0236	138	210	60	.0500
9	12	71	127	27	.0245	75	123	31	.0477	12	17	135	225	58	.0238	142	218	64	.0486
9	13	73	134	29	.0217	78	129	34	.0478	12	18	139	233	62	.0239	146	226	68	.0474
9	14	76	140	32	.0228	81	135	37	.0478	12	19	143	241	66	.0240	150	234	72	.0463
9	15	79	146	35	.0238	84	141	40	.0478	12	20	147	249	70	.0241	155	241	77	.0493
9	16	82	152	38	.0247	87	147	43	.0477	12	21	151	257	74	.0242	159	249	81	.0481
9	17	84	159	40	.0223	90	153	46	.0476	12	22	155	265	78	.0242	163	257	85	.0471
9	18	87	165	43	.0231	93	159	49	.0475	12	23	159	273	82	.0243	168	264	90	.0496
9	19	90	171	46	.0239	96	165	52	.0474	12	24	163	281	86	.0243	172	272	94	.0486
9	20	93	177	49	.0245	99	171	55	.0473	12	25	167	289	90	.0243	176	280	98	.0475
9	21	95	184	51	.0225	102	177	58	.0472	13	13	136	215	46	.0221	142	209	51	.0454
9	22	98	190	54	.0231	105	183	61	.0471	13	14	141	223	51	.0241	147	217	56	.0472
9	23	101	196	57	.0237	108	189	64	.0470	13	15	145	232	55	.0232	152	225	61	.0489
9	24	104	202	60	.0243	111	195	67	.0469	13	16	150	240	60	.0250	156	234	65	.0458
9	25	107	208	63	.0249	114	201	70	.0468	13	17	154	249	64	.0240	161	242	70	.0472
10	10	78	132	24	.0216	82	128	28	.0446	13	18	158	258	68	.0232	166	250	75	.0485
10	11	81	139	27	.0215	86	134	32	.0493	13	19	163	266	73	.0247	171	258	80	.0497
10	12	84	146	30	.0213	89	141	35	.0465	13	20	167	275	77	.0238	175	267	84	.0470
10	13	88	152	34	.0247	92	148	38	.0441	13	21	171	284	81	.0231	180	275	89	.0481
10	14	91	159	37	.0242	96	154	42	.0478	13	22	176	292	86	.0243	185	283	94	.0491
10	15	94	166	40	.0238	99	161	45	.0455	13	23	180	301	90	.0236	189	292	98	.0467
10	16	97	173	43	.0234	103	167	49	.0487	13	24	185	309	95	.0247	194	300	103	.0476
10	17	100	180	46	.0230	106	174	52	.0465	13	25	189	318	99	.0240	199	308	108	.0485
10	18	103	187	49	.0226	110	180	56	.0493	14	14	160	246	56	.0249	166	240	61	.0469
10	19	107	193	53	.0250	113	187	59	.0472	14	15	164	256	60	.0229	171	249	66	.0466

1-tail 2-tail		$\alpha = 0.025$ $\alpha = 0.05$				$\alpha = 0.05$ $\alpha = 0.10$				1-tail 2-tail		$\alpha = 0.025$ $\alpha = 0.05$				$\alpha = 0.05$ $\alpha = 0.10$			
m	n	W		d	P	W		d	P	m	n	W		d	P	W		d	P
14	16	169	265	65	.0236	176	258	72	.0463	17	24	282	432	130	.0239	294	420	141	.0492
14	17	174	274	70	.0242	182	266	78	.0500	17	25	288	443	136	.0238	300	431	147	.0480
14	18	179	283	75	.0247	187	275	83	.0495	18	18	270	396	100	.0235	280	386	109	.0485
14	19	183	293	79	.0230	192	284	88	.0489	18	19	277	407	107	.0246	287	397	116	.0490
14	20	188	302	84	.0235	197	293	93	.0484	18	20	283	419	113	.0238	294	408	123	.0495
14	21	193	311	89	.0239	202	302	98	.0480	18	21	290	430	120	.0247	301	419	130	.0499
14	22	198	320	94	.0243	207	311	103	.0475	18	22	296	442	126	.0240	307	431	136	.0474
14	23	203	329	99	.0247	212	320	108	.0471	18	23	303	453	133	.0248	314	442	143	.0478
14	24	207	339	103	.0233	218	328	114	.0498	18	24	309	465	139	.0240	321	453	150	.0481
14	25	212	348	108	.0236	223	337	119	.0492	18	25	316	476	146	.0248	328	464	157	.0484
15	15	184	281	65	.0227	192	273	73	.0488	19	19	303	438	114	.0248	313	428	123	.0482
15	16	190	290	71	.0247	197	283	78	.0466	19	20	309	451	120	.0234	320	440	130	.0474
15	17	195	300	76	.0243	203	292	84	.0485	19	21	316	463	127	.0236	328	451	138	.0494
15	18	200	310	81	.0239	208	302	89	.0465	19	22	323	475	134	.0238	335	463	145	.0486
15	19	205	320	86	.0235	214	311	95	.0482	19	23	330	487	141	.0240	342	475	152	.0478
15	20	210	330	91	.0232	220	320	101	.0497	19	24	337	499	148	.0241	350	486	160	.0496
15	21	216	339	97	.0247	225	330	106	.0478	19	25	344	511	155	.0243	357	498	167	.0488
15	22	221	349	102	.0243	231	339	112	.0492	20	20	337	483	128	.0245	348	472	138	.0482
15	23	226	359	107	.0239	236	349	117	.0474	20	21	344	496	135	.0241	356	484	146	.0490
15	24	231	369	112	.0235	242	358	123	.0486	20	22	351	509	142	.0236	364	496	154	.0497
15	25	237	378	118	.0248	248	367	129	.0499	20	23	359	521	150	.0246	371	509	161	.0478
16	16	211	317	76	.0234	219	309	84	.0469	20	24	366	534	157	.0242	379	521	169	.0484
16	17	217	327	82	.0243	225	319	90	.0471	20	25	373	547	164	.0237	387	533	177	.0490
16	18	222	338	87	.0231	231	329	96	.0473	21	21	373	530	143	.0245	385	518	154	.0486
16	19	228	348	93	.0239	237	339	102	.0474	21	22	381	543	151	.0249	393	531	162	.0482
16	20	234	358	99	.0247	243	349	108	.0475	21	23	388	557	158	.0238	401	544	170	.0478
16	21	239	369	104	.0235	249	359	114	.0475	21	24	396	570	166	.0242	410	556	179	.0497
16	22	245	379	110	.0242	255	369	120	.0476	21	25	404	583	174	.0245	418	569	187	.0492
16	23	251	389	116	.0248	261	379	126	.0476	22	22	411	579	159	.0247	424	566	171	.0491
16	24	256	400	121	.0238	267	389	132	.0476	22	23	419	593	167	.0244	432	580	179	.0477
16	25	262	410	127	.0243	273	399	138	.0476	22	24	427	607	175	.0242	441	593	188	.0486
17	17	240	355	88	.0243	249	346	97	.0493	22	25	435	621	183	.0240	450	606	197	.0494
17	18	246	366	94	.0243	255	357	103	.0479	23	23	451	630	176	.0249	465	616	189	.0499
17	19	252	377	100	.0243	262	367	110	.0499	23	24	459	645	184	.0242	474	630	198	.0497
17	20	258	388	106	.0242	268	378	116	.0485	23	25	468	659	193	.0246	483	644	207	.0495
17	21	264	399	112	.0242	274	389	122	.0473	24	24	492	684	193	.0241	507	669	207	.0486
17	22	270	410	118	.0241	281	399	129	.0490	24	25	501	699	202	.0241	517	683	217	.0496
17	23	276	421	124	.0240	287	410	135	.0477	25	25	536	739	212	.0247	552	723	227	.0497